ELEMENTARY LINEAR ALGEBRA – SET 5

Vector spaces, linear independence, basis

1. Show that the vectors $v_1 = (1, 1, 1)$, $v_2 = (1, 2, 3)$ and $v_3 = (2, -1, 1)$ are linearly independent in $\mathbb{R}^3$.

2. Show that the vectors $v_1 = (1, 2)$, $v_2 = (2, 1)$ and $v_3 = (1, 0)$ are linearly dependent in $\mathbb{R}^2$.

3. Check if the vectors $v_1 = (1, 2, 0)$, $v_2 = (-2, 1, 1)$, $v_3 = (-1, 3, 1)$ are linearly independent in $\mathbb{R}^3$.

4. Check if the following vectors span the space $\mathbb{R}^3$:

   (a) $v_1 = (1, 1, 0)$, $v_2 = (1, 2, 0)$, $v_3 = (0, 0, 1)$,

   (b) $v_1 = (1, 2, 3)$, $v_2 = (-1, -1, -3)$, $v_3 = (-1, 1, -3)$,

   (c) $v_1 = (1, 0, 1)$, $v_2 = (0, 1, 1)$, $v_3 = (1, 1, 0)$, $v_4 = (1, 1, 1)$.

5. Check if vectors from Problem 4 form a basis in $\mathbb{R}^3$.

6. Check if the following sets of vectors form a subspace of $\mathbb{R}^3$:

   (a) $W = \{t(1, 1, 1) + (0, 1, 2) : t \in \mathbb{R}\}$

   (b) $W = \{(x, y, z) : x + y = z\}$

   (c) $W = \{(x, y, z) : x + y + z = 1\}$

7. Determine the dimension of each of the following subspaces of $\mathbb{R}^4$:

   (a) $W = \{(x, y, z, t) : x + y = t\}$

   (b) $W = \{(x, y, z, t) : t = x + y, z = x - y\}$

   (c) $W = \{(x, y, z, t) : x = y = z = t\}$

8. Find the coordinates of the vector $v = (3, 2, 3)$ in the following bases in $\mathbb{R}^3$:

   (a) $B = \{(1, 0, 0), (1, 1, 0), (1, 1, 1)\}$

   (b) $B = \{(1, 1, 0), (1, 1, 1), (0, 1, 1)\}$.

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